Eur. Phys. J. B **36**, 215–220 (2003)
DOI: 10.1140/epjb/e2003-00338-5

THE EUROPEAN
PHYSICAL JOURNAL B

Periodic instantons and domain structure in a ferromagnetic film

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Received 19 May 2003 / Received in final form 6 August 2003
Published online 8 December 2003 –

Abstract. We in this paper study periodic instantons and domain structures in a theoretical film consisting of biaxial-anisotropic ferromagnets. In a proper approximation the equation of motion of the magnetization vector as a space-time function in the film is reduced to the 1+2-dimensional sine-Gordon field equation in strong anisotropy limit. Static periodic instantons, which are solutions of Euclidean field equantion, and various new domain structures are obtained analytically. We also investigate the energy density and stability of the periodic instantons.

PACS. 05.45.Yv Solitons – 75.70.Kw Domain structure (including magnetic bubbles)

1 Introduction

Macroscopic magnetization tunneling is a subject which has been investigated extensively and is of growing interest [1–6]. When either a giant spin or (in bulk material) a domain wall tunnels between degenerate states, we have the situation of macroscopic quantum coherence (MQC) [1–6]. Until now only magnetic molecular clusters, for instance the octanuclear iron oxo-hydroxo cluster Fe_8, have been the most promising candidates to observe MQC [7,8]. Quantum tunneling in this case is dominated by the instanton configuration which is the solution of Euclidean classical field equation with finite action [9–14]. The instanton technique is a powerful tool in the evaluation of the tunneling rate [9–14]. However, there are very few theories which permit an explicit calculation and investigation of such classical field configurations i.e instantons except in the case of 1-dimension where the explicit instanton solution is available and has been widely used to study the tunneling effects in various branches of physics, especially, the macroscopic magnetization tunneling in a single-domain magnet [7,8]. It is certainly of interest to extend the investigation of macroscopic quantum tunneling in a single-domain magnet to the tunneling in a field model and find the instanton configurations in higher dimensions. The main goal of the present paper is to obtain the static instanton configurations (domain walls), the time dependent versions of which are responsible for quantum tunneling [13,14], in the theoretical film consisting of mesoscopic magnets.

The domain structures which are the static solutions of real time field equation in the film have also attracted considerable interests [15–18] recently motivated by the possible applications in the semiconductor microelectronics technology and the development of devices based on the giant magnetoresistance effect.

This paper is of two-fold. In the first part the instanton configurations are investigated and various domain structures are given in the second part. Particularly in Section 2 we introduce the film-model with magnetization vector lying in the film-plane in the strong anisotropy limit which is seen to be described by the 1+2-dimensional sine-Gordon field equation. Section 3 is devoted to the static periodic instanton configurations which minimize the Euclidean field action and are seen to be the domain-wall solutions in the system considered. The stability of static periodic instantons is analyzed in Section 4. Finally in Section 5 we present various domain structures which are static solutions of (real time) field equation.

2 Model

We begin with the Hamiltonian operator of 2-dimensional simple cubic lattice consisting of biaxial anisotropic ferromagnets described by the spin operator $\widehat{S}$,

$$\hat{H} = -J\sum_{\langle i,j\rangle}\hat{S}_i.\hat{S}_j + K_1\sum_i\left(\hat{S}_i^z\right)^2 + K_2\sum_i\left(\hat{S}_i^y\right)^2, \quad (2.1)$$

where $J\,(>0)$ is the exchange constant and $K_1 > K_2 > 0$ are the anisotropy energies. The ferromagnet at each lattice site possesses a XY easy plane with a X easy axis. From the Heisenberg equation of motion for the spin operator of the ferromagnet on the kth site

$$\frac{d}{dt}\hat{S}_k = \frac{1}{i\hbar}\left[\hat{S}_k, \hat{H}\right], \quad (2.2)$$

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and following reference [19] to treat the giant spin as a classical vector [20,21] such that $\hat{S}_k \rightarrow \vec{S}(x,y,t)$ the differential equations of the spin vector in the spherical coordinates $\vec{S} = S(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ is found as

$$\frac{\partial\theta}{\partial t} = -aS\left[2\cos\theta\left(\frac{\partial\theta}{\partial x}\right)\left(\frac{\partial\varphi}{\partial x}\right) + 2\cos\theta\left(\frac{\partial\theta}{\partial y}\right)\left(\frac{\partial\varphi}{\partial y}\right) + \sin\theta\left(\frac{\partial^2\varphi}{\partial x^2}\right) + \sin\theta\left(\frac{\partial^2\varphi}{\partial y^2}\right)\right] + \frac{\kappa_2}{2}S\sin\theta\sin 2\varphi,$$

$$\sin\theta\left(\frac{\partial\varphi}{\partial t}\right) = aS\left[\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} - \frac{1}{2}\sin 2\theta\left(\frac{\partial\varphi}{\partial x}\right)^2 - \frac{1}{2}\sin 2\theta\left(\frac{\partial\varphi}{\partial y}\right)^2\right] + \frac{\kappa_1}{2}S\sin 2\theta - \frac{\kappa_2}{2}S\sin 2\theta\sin^2\varphi, \quad (2.3)$$

where the constants a and $\kappa_{1,2}$ are given by

$$a = \frac{2J}{\hbar}, \quad \kappa_1 = \frac{2K_1}{\hbar}, \quad \kappa_2 = \frac{2K_2}{\hbar},$$

and x and y denote the dimensionless spatial coordinates measured in the unit of lattice constant. For strong uniaxial anisotropy, i.e. $\kappa_1 \gg \kappa_2$ (this limit may not be the case for molecular cluster Fe_8), the spin vectors are forced to rotate in the XY easy plane i.e. in plane of film and we may assume that $\theta = \frac{\pi}{2} - \delta$ where δ denotes a small perturbation angle. Up to the first order of the small quantity δ and noticing again the strong uniaxial anisotropy $\kappa_1 \gg a$ we obtain

$$\frac{\partial\delta}{\partial t} = aS\frac{\partial^2\varphi}{\partial x^2} + aS\frac{\partial^2\varphi}{\partial y^2} - \frac{\kappa_2}{2}S\sin 2\varphi,$$
$$\frac{\partial\varphi}{\partial t} = \kappa_1 S\delta. \quad (2.4)$$

The equation of motion for the angle φ is seen to be

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} - \frac{\partial^2\varphi}{\partial\tilde{t}^2} = \eta\sin 2\varphi, \quad (2.5)$$

where $\tilde{t} = S\sqrt{a\kappa_1}\; t$ is the dimensionless time and the constant $\eta = \frac{k_2}{2a}$ is dimensionless. From now on we drop the over head "$\sim$" on time t and the dimensionless time is understood.

The Lagrangian density can be written as

$$\pounds = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial y}\right)^2 - V(\varphi),$$
$$V(\varphi) = \frac{1}{g^2}(1-\cos 2\varphi), \quad (2.6)$$

with $g = \sqrt{\frac{2}{\eta}}$. The canonical momentum density is defined by

$$\Pi = \frac{\partial\pounds}{\partial\frac{\partial\varphi}{\partial t}} = \frac{\partial\varphi}{\partial t}, \quad (2.7)$$

and we have the Hamiltonian density

$$H = \frac{1}{2}\Pi^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial y}\right)^2 + V(\varphi). \quad (2.8)$$

3 Static instantons (domain walls) of Euclidean field equation

Instanton method plays a central role in the investigation of quantum tunneling. Following references [13,22] we derive the periodic instantons of two-dimension.

The Euclidean Lagrangian is

$$\pounds_e = \frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial y}\right)^2 + V(\varphi), \quad (3.1)$$

where $\tau = it$ denotes the imaginary time. The static Euclidean field equation is seen to be

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \eta\sin 2\varphi. \quad (3.2)$$

We look for trial solution $\varphi(\xi)$ with $\xi = x + uy$,where u is a constant. Then we have

$$\left(1+u^2\right)\frac{d^2\varphi}{d\xi^2} = \eta\sin 2\varphi. \quad (3.3)$$

The integrated equation of motion is seen to be

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 - \frac{1}{(1+u^2)}V(\varphi) = -E, \quad (3.4)$$

with the classical energy $E \geq 0$ being a constant of integration. We define $k^2 = 1 - \frac{(1+u^2)g^2E}{2}$ and obtain

$$\left(\frac{d\varphi}{d\xi}\right)^2 = \frac{4}{(1+u^2)\,g^2}\left(k^2 - \cos^2\varphi\right). \quad (3.5)$$

The solution for the energy parameter E confined to a region $0 < E < \frac{2}{(1+u^2)g^2}$ is found as

$$\varphi_s(\xi) = \arccos\left[ksn\left(-\frac{2}{g\sqrt{1+u^2}}\xi, k\right)\right], \quad (3.6)$$

where sn, cn, dn, ... are Jacobian elliptic functions with modulus k. The definition and properties of the Jacobian elliptic functions are given in the Appendix. $\varphi_s(\xi) = \varphi_s(\xi + L)$ is a periodic function with period L such that

$$\frac{2}{g\sqrt{1+u^2}}L = 4nK(k), \quad (3.7)$$

where $K(k)$ is the complete elliptic integral of the first kind and $n = 1, 2, 3...$ We demand that $\varphi_s(x,y)$be periodic with periods L_x and L_y along the x and y-axis respectively, i.e.,

$$\varphi_s(x,y) = \varphi_s(x + L_x, y) = \varphi_s(x, y + L_y), \quad (3.8)$$

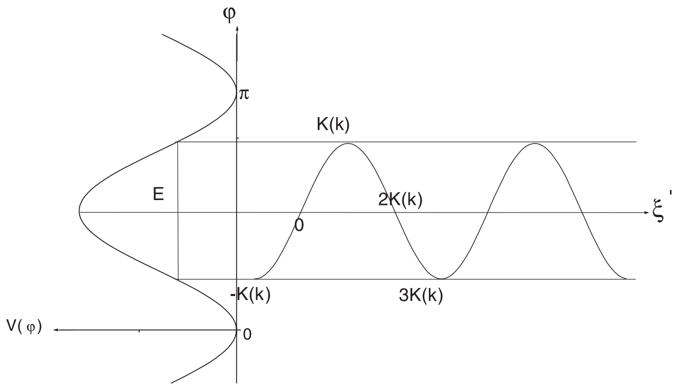

Fig. 1. Periodic domain-wall configuration.

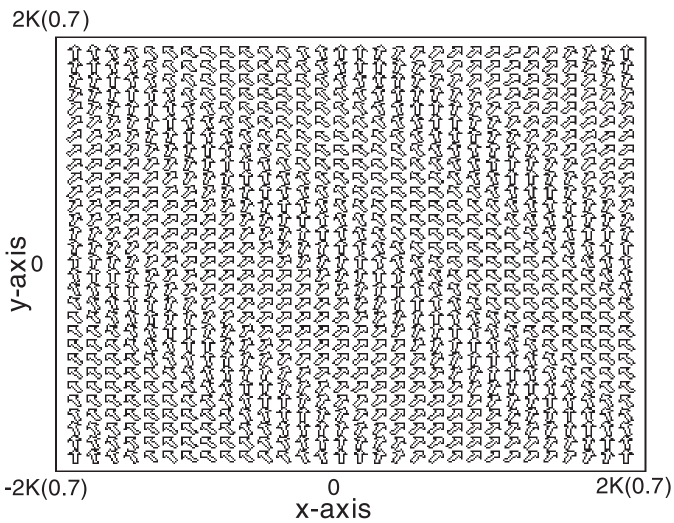

Fig. 2. Spatial distribution of magnetization vector for domain-wall solution (3.6) of $k = 0.7$ with the units of the x and y-axis being $g\sqrt{1+u^2}/2$ and $g\sqrt{1+u^2}/2|u|$ respectively.

which leads to

$$\frac{2L_x}{g\sqrt{1+u^2}} = 2K(k),$$

$$\frac{2|u|L_y}{g\sqrt{1+u^2}} = 2K(k). \tag{3.9}$$

The periodic potential and the static instanton $\varphi_s(\xi)$, which is also known as sphaleron in field theory [13,22], are shown in Figure 1. The magnetization vector as a function of spatial coordinates is plotted in Figure 2 with the modulus $k = 0.7$.

In the limit $k \to 0$ ($E \to \frac{2}{(1+u^2)g^2}$, namely, the energy approaches the top of the barrier), the solution tends to a trivial configuration

$$\varphi_s^t = \frac{\pi}{2}. \tag{3.10}$$

In the limit $k \to 1 (E \to 0)$, the solution becomes

$$\varphi_s^0(x,y) = \arccos\left[\tanh\left(-\frac{2}{g}\frac{(x+uy)}{\sqrt{1+u^2}}\right)\right], \tag{3.11}$$

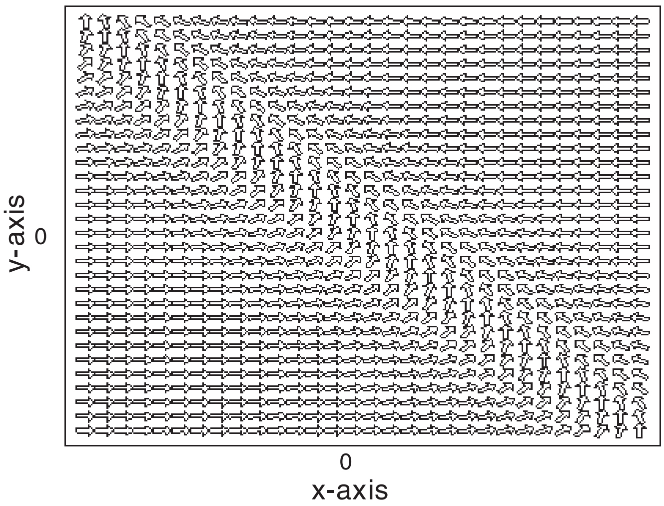

Fig. 3. Spatial distribution of magnetization vector for domain-wall solution (3.11) of $k = 1.0$.

which is seen to be the familiar domain wall configuration shown in Figure 3.

The energy density of a static field configuration in a strip of one period length and unit width can be obtained from the static energy functional [22,23]

$$\varepsilon[\varphi_s] = \int_{-K(k)}^{K(k)} d\xi' \left[\frac{1}{2}\left(\frac{\partial\varphi_s}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi_s}{\partial y}\right)^2 + V(\varphi_s)\right]$$

$$= \frac{4k'^2}{g^2}K(k) + \frac{8}{g^2}\left[E(k) - k'^2 K(k)\right], \tag{3.12}$$

where $\xi' = \frac{2}{g\sqrt{1+u^2}}\xi$, $k' = \sqrt{1-k^2}$ and $E(k)$ denotes the complete elliptic integral of the second kind. Using the expansion of complete elliptic integrals $K(k)$ and $E(k)$ in power series of modulus

$$K(k) = \frac{\pi}{2}\left[1 + \frac{1}{4}k^2 + \cdots\right], \quad k \to 0,$$

$$E(k) = \frac{\pi}{2}\left[1 - \frac{1}{4}k^2 + \cdots\right], \quad k \to 0, \tag{3.13}$$

$$K(k) = \ln\frac{4}{k'} + \frac{1}{4}\left(\ln\frac{4}{k'} - 1\right)(k')^2 + \cdots, \quad k \to 1,$$

$$E(k) = 1 + \frac{1}{2}\left(\ln\frac{4}{k'} - \frac{1}{2}\right)(k')^2 + \cdots, \quad k \to 1, \tag{3.14}$$

we find the corresponding energy densities

$$\varepsilon\left[\varphi_s^t\right] = \frac{2\pi}{g^2}, \tag{3.15}$$

$$\varepsilon\left[\varphi_s^0\right] = \frac{8}{g^2}, \tag{3.16}$$

for the trivial configurations φ_s^t and φ_s^0 respectively. The static instantons (called sphalerons [13,22]) play an important role in the barrier transition. The barrier transition rate at temperature T per unit area in the film can be evaluated in terms of the energy density of sphaleron such that $\Gamma \sim e^{-\frac{\varepsilon[\varphi_s]}{K_B T}}$ known as Arrhenius law, where K_B is the Boltzmann constant and the time-dependent instantons are responsible for the quantum tunneling [14].

4 Stability of the periodic (static) instantons

It is interesting to present a general criterion of the stability of the static configuration $\varphi_c(x, y)$. To this end we study time-dependent equation to obtain the normal modes of small oscillation around the classical solution such that

$$\varphi(x,y,t) = \varphi_c(x,y) + \sum_k \psi_k(x,y)\, e^{i\omega_k t}, \tag{4.1}$$

where $\psi_k(x,y)$ denotes perturbation field. Substituting this form into the time-dependent equation (2.5), we obtain the two-dimensional Schrödinger like equation

$$\left\{-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + V''[\varphi_c(x,y)]\right\}\psi_k(x,y) = \omega_k^2 \psi_k(x,y), \tag{4.2}$$

with periodic condition

$$\psi_k(x,y) = \psi_k(x+L_x, y) = \psi_k(x, y+L_y). \tag{4.3}$$

The periods are obtained from equation (3.9) as

$$L_x = gK(k)\sqrt{1+u^2}, \quad L_y = \frac{g}{|u|}K(k)\sqrt{1+u^2}. \tag{4.4}$$

For the constant solution $\varphi_s^t = \frac{\pi}{2}$, equation (4.2) is reduced to

$$\left\{-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{4}{g^2}\right\}\psi_k(x,y) = \omega_k^2 \psi_k(x,y). \tag{4.5}$$

It is found that

$$\psi_{k_x,k_y} \propto \frac{\sin\left(\frac{2\pi}{L_x}k_x x\right)\sin\left(\frac{2\pi}{L_y}k_y y\right)}{\cos\left(\frac{2\pi}{L_x}k_x x\right)\cos\left(\frac{2\pi}{L_y}k_y y\right)}, \tag{4.6}$$

with $k_x, k_y = 0, 1, 2...$ and

$$\omega_{k_x,k_y}^2 = -\frac{4}{g^2} + \frac{4\pi^2}{L_x^2}k_x^2 + \frac{4\pi^2}{L_y^2}k_y^2. \tag{4.7}$$

Using the expansion of complete elliptic integral $K(k)$ in power series of the modulus k for $k \to 0$, we obtain

$$\omega_{k_x,k_y}^2 = -\frac{4}{g^2}\left[1 - \frac{k_x^2}{(1+u^2)} - \frac{u^2 k_y^2}{(1+u^2)}\right], \tag{4.8}$$

which is negative for $k_x = k_y = 0$ and the trivial configuration φ_s^t is unstable. For the periodic configuration (3.6)

$$V''[\varphi_s(x,y)] = -\frac{4}{g^2}\left[1 - 2k^2 sn^2\left(-\frac{2}{g}\frac{(x+uy)}{\sqrt{1+u^2}}, k\right)\right], \tag{4.9}$$

we obtain the discrete eigenfunctions

$$\psi_1(x,y) = cn\left(-\frac{2}{g}\frac{(x+uy)}{\sqrt{1+u^2}}, k\right),$$
$$\psi_2(x,y) = dn\left(-\frac{2}{g}\frac{(x+uy)}{\sqrt{1+u^2}}, k\right),$$
$$\psi_3(x,y) = sn\left(-\frac{2}{g}\frac{(x+uy)}{\sqrt{1+u^2}}, k\right), \tag{4.10}$$

and corresponding eigenvalues

$$\omega_1^2 = 0, \ \omega_2^2 = \frac{4}{g^2}\left(k^2 - 1\right), \ \omega_3^2 = \frac{4}{g^2}k^2. \tag{4.11}$$

The second one is negative and the periodic, static instanton equation (3.6) is unstable too, while the domain wall solution equation (3.11) corresponding to $k \to 1$ is stable seen from the fact that the negative mode ω_2^2 merges to the zero mode ω_1^2 in this case. The stability behavior of two-dimensional sphalerons is the same as that of one-dimensional case [13].

5 Domain structures

Static solutions of real time field equation (2.5) may be also of interest. We scale the spatial coordinates by $\widetilde{x} = \sqrt{\eta}x$, $\widetilde{y} = \sqrt{\eta}y$ and the equation becomes

$$\frac{\partial^2\varphi}{\partial\widetilde{x}^2} + \frac{\partial^2\varphi}{\partial\widetilde{y}^2} = \sin 2\varphi. \tag{5.1}$$

From now on we drop the over head "$\sim$" on the dimensionless spatial coordinates with the new scale understood. Assuming $\varphi = 2\arctan\left[\frac{X(x)}{Y(y)}\right]$ [24] we have

$$X'' = 2\alpha X^3 + (2-\beta)X,$$
$$Y'' = 2\alpha Y^3 + \beta Y, \tag{5.2}$$

here, α, β are arbitrary constants.

(1) $\alpha > 0$, $\beta < 0$, we obtain the breather like configuration

$$\varphi(x,y) = 2\arctan\left\{\pm\sqrt{\frac{2-\beta}{-\beta}}\frac{\sin\left[\sqrt{-\beta}y\right]}{\sinh\left[\sqrt{2-\beta}x\right]}\right\}, \tag{5.3}$$

with the magnetization vector plotted in Figure 4.

(2) $\alpha > 0$, $\beta = 1$, we have

$$\varphi(x,y) = 2\arctan\left\{\pm\frac{\cot\left[\sqrt{\frac{1}{2}}x\right]}{\cot\left[\sqrt{\frac{1}{2}}y\right]}\right\}, \tag{5.4}$$

with the spatial distribution of magnetization vector shown in Figure 5.

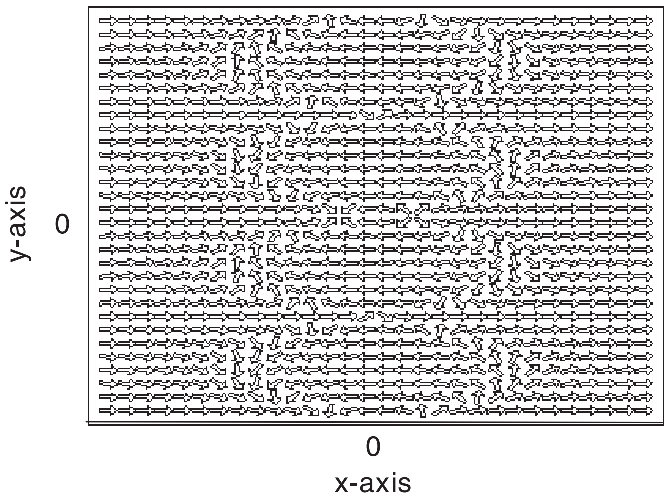

Fig. 4. Spatial distribution of magnetization vector for solution (5.3) of $\beta = -1$.

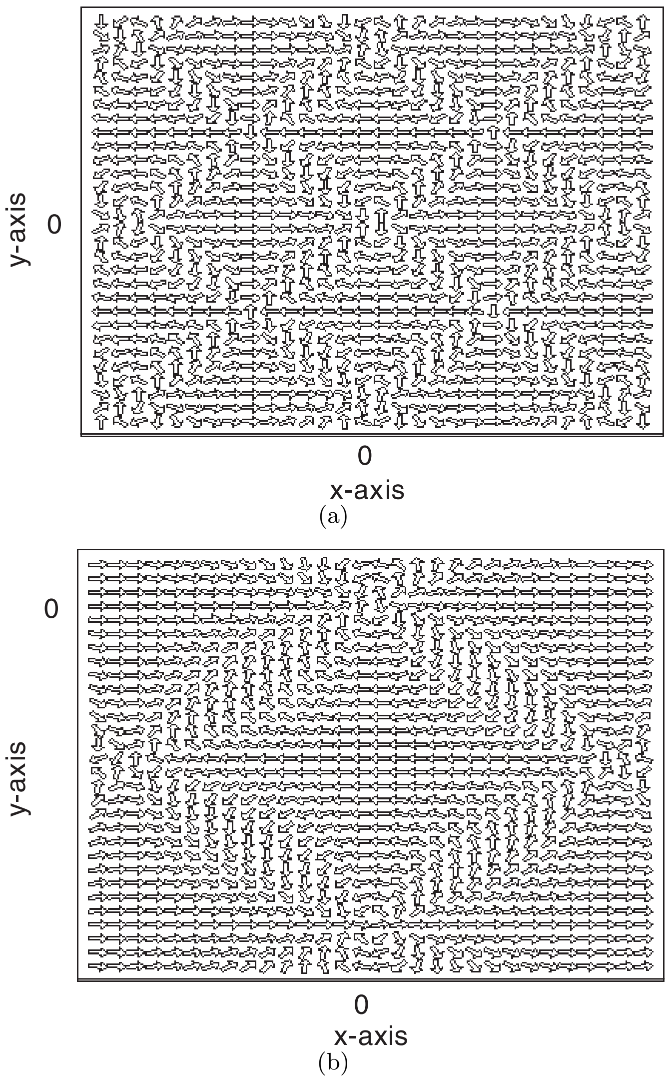

Fig. 5. Spatial distribution of magnetization vector for solution (5.4). (a) multi-period, (b) one period.

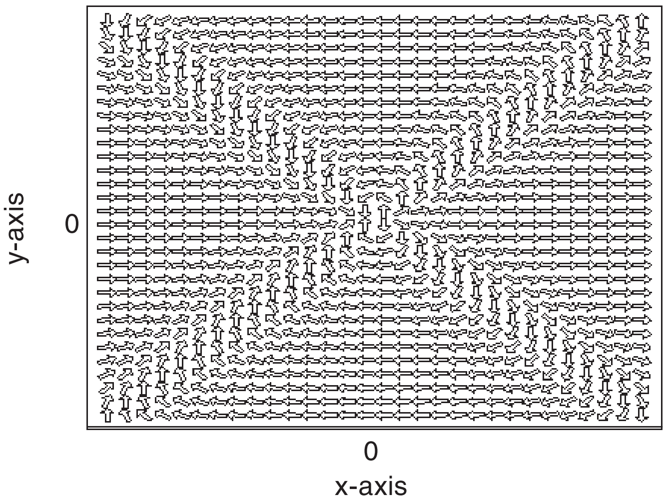

Fig. 6. Spatial distribution of magnetization vector for solution (5.5) of $\beta = 1$.

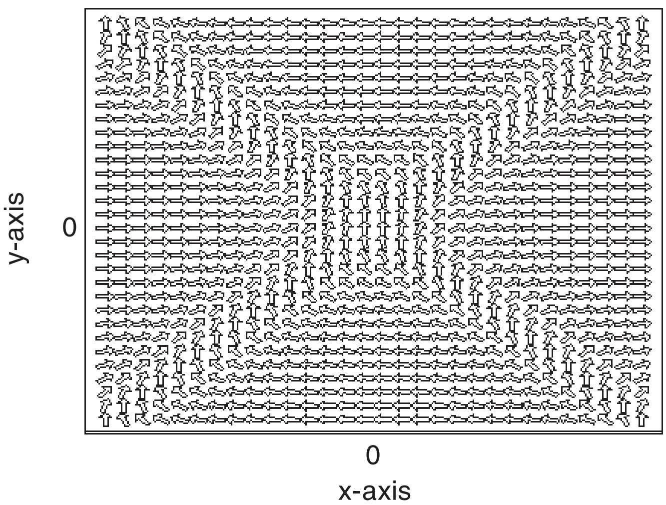

Fig. 7. Spatial distribution of magnetization vector for solution (5.6) of $\beta = 1$.

(3) $\alpha > 0$, $0 < \beta < 2$, we have

$$\varphi(x,y) = 2\arctan\left\{\pm\sqrt{\frac{2-\beta}{\beta}}\frac{\sinh\left[\sqrt{\beta}y\right]}{\sinh\left[\sqrt{2-\beta}x\right]}\right\}, \quad (5.5)$$

and the magnetization vector is plotted in Figure 6.

(4) $\alpha < 0$, $0 < \beta < 2$, the solution of domain structure is

$$\varphi(x,y) = 2\arctan\left\{\pm\sqrt{\frac{2-\beta}{\beta}}\frac{\cosh\left[\sqrt{\beta}y\right]}{\cosh\left[\sqrt{2-\beta}x\right]}\right\}, \quad (5.6)$$

with the spatial distribution of magnetization vector shown in Figure 7. The solutions equations (5.3, 5.4) are periodic configurations. It may be worth while to emphasize that the domain structures obtained here are valid in an infinite film without boundary. For a practical film of finite size a proper boundary condition is required such that the magnets at boundary should be parallel to the boundary edge in order to remove the possible net magnetic charge so that having lower energy.

6 Conclusion

Periodic static instantons and domain structures in the ferromagnetic film are obtained analytically. We pay special attention to the periodic domain-wall configurations (sphalerons) which are static solutions of Euclidean field equation and are responsible for the barrier transition in the two-dimensional field model. The energy densities equations (3.12, 3.15) and (3.16) can be directly used to evaluate the barrier transition rate in the corresponding field model. However the study of the quantum tunneling effects is beyond the scope of the present paper. Besides the application in the barrier transition the new domain-wall configurations and domain structures of their own are of theoretical and practical interests.

This work was supported by the Nature Science Foundation of China under Grant No. 10075032.

Appendix: Jacobian elliptic functions

The Jacobian elliptic function $sn(p,k)$ of modulus k ($0 < k < 1$) is defined in the integral form,

$$p = \int_0^q \frac{dt}{\sqrt{(1-t^2)\,(1-k^2t^2)}}, \tag{A.1}$$

where $q = sn\,(p,k)$ or $p = sn^{-1}\,(q)$. The functions $cn\,(p,k)$ and $dn\,(p,k)$ are defined by the following relations,

$$cn\,(p,k) = \sqrt{1 - sn^2\,(p,k)}, \tag{A.2}$$

$$dn\,(p,k) = \sqrt{1 - k^2 sn^2\,(p,k)}. \tag{A.3}$$

More properties of these functions can be found in reference [25].

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